

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

DIM5068 – MATHEMATICAL TECHNIQUES 2

(T71)

25 OCTOBER 2017

02.30 pm – 04.30 pm

(2 Hours)

INSTRUCTIONS TO STUDENT

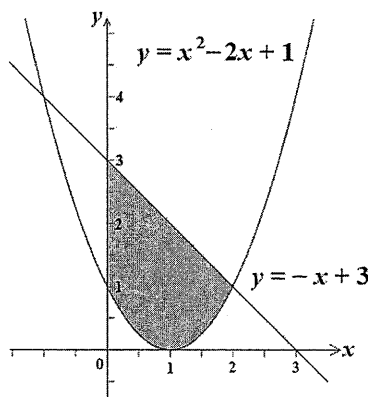
1. This question paper consists of **2** pages with **4** questions.
2. Attempt **ALL FOUR** questions. The marks distribution for each question is given.
3. Write your answers in the answer booklet provided.
4. Key formulae are given in the Appendix.

Question 1

- a) Given $Z_1 = a + 4i$ and $Z_2 = 1 - bi$ are two complex numbers such that, $Z_1 + Z_2 = 1$. Find the values of a and b . (4 marks)
- b) Find the complex zeros of the polynomial functions, $f(x) = x^3 - 5x^2 + 2x - 10$ and rewrite the function in factored form. (6 marks)

[TOTAL 10 MARKS]**Question 2**

- a) Differentiate $y = 7x^7 + 7\sec x + 7\ln x - 7$. (2 marks)
- b) Given the function of $y = \frac{3}{2(9x+3)^2}$.
- Find the first derivative by using chain rule. (4 marks)
 - Show that $\frac{dy}{dx} = -1$ when $x = 0$. (2 marks)
- c) For the function $f(x) = 2x^3 + 4x^2 + 2x$,
- find the critical number(s). (4 marks)
 - identify the intervals on which f is increasing or decreasing. (3 marks)
 - determine the maximum and/or the minimum value(s). (2 marks)
- d) Integrate $\int (2x + 8e^x - 5) dx$. (2 marks)
- e) By using substitution rule, integrate $\int \frac{6x^3}{(x^4 - 7)^{10}} dx$. (5 marks)
- f) Find the area of a farm as shown by the shaded region below, which is bounded by $y = x^2 - 2x + 1$, $y = -x + 3$ and the sides from $x = 0$ until $x = 2$. (6 marks)

**[TOTAL 30 MARKS]****Continued...**

Question 3

a) Solve for y , given that the first order differential equation is $\frac{dy}{dx} = \frac{12x^2 + 4x}{y}$, by using the

Separable Method.

(5.5 marks)

b) Given the differential equation $\frac{dy}{dx} + \frac{y}{x} = \cos x$.

i. Identify the $p(x)$ and $q(x)$. (2 marks)

ii. Calculate the integrating factor, μ . (2 marks)

iii. Find μy given $\mu y = \int \mu q(x) dx$. [Hint: Use integration by parts] (5.5 marks)

iv. From your answer in (iii), determine the solution of the initial value problem if $y(0) = 1$.

(2 marks)

v. State the general solution of y .

(1 mark)

c) Given the non-homogeneous differential equation $y'' - 8y' + 15y = 75x - 25$.

i. Determine the complementary solution, y_c . (4 marks)

ii. Compute the particular solution, y_p . (8 marks)

[TOTAL 30 MARKS]

Question 4

a) Given that $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 12\mathbf{i} - 5\mathbf{j}$. Find,

i. $3\mathbf{a} + 2\mathbf{b}$. (4 marks)

ii. a unit vector that has the same direction as $\mathbf{a}$. (3 marks)

iii. a vector orthogonal to both $\mathbf{a}$ and $\mathbf{b}$. (4 marks)

b) Find a vector orthogonal to the plane through points $A(2,0,-4)$, $B(4,5,8)$ and $C(3,2,7)$. Then find the area of the triangle ABC . (12 marks)

c) Find the vector equation, parametric equation and symmetric equation for line through point $(1,-2,4)$ and is parallel to vector $\langle 4,-6,8 \rangle$. (7 marks)

[TOTAL 30 MARKS]

End of Page.

APPENDIX – KEY FORMULA

Derivatives: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

Differentiation Rules

General Formulae

$$1. \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \quad 2. \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$3. \frac{d}{dx}(x^n) = nx^{n-1} \quad 4. \frac{d}{dx}[f(u)] = \frac{dy}{du} \cdot \frac{du}{dx}$$

Exponential and Logarithmic Functions

$$1. \frac{d}{dx}(e^x) = e^x \quad 2. \frac{d}{dx}(a^x) = a^x \ln a$$

$$3. \frac{d}{dx}(\ln x) = \frac{1}{x} \quad 4. \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Trigonometric Functions

$$1. \frac{d}{dx}(\sin x) = \cos x \quad 2. \frac{d}{dx}(\cos x) = -\sin x$$

$$3. \frac{d}{dx}(\tan x) = \sec^2 x \quad 4. \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$5. \frac{d}{dx}(\sec x) = \sec x \tan x \quad 6. \frac{d}{dx}(\cot x) = -\csc^2 x$$

Table of Integrals

$$1. \int u \, dv = uv - \int v \, du \quad 2. \int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$3. \int \frac{du}{u} = \ln|u| + C \quad 4. \int e^u \, du = e^u + C$$

$$5. \int \sin u \, du = -\cos u + C \quad 6. \int \cos u \, du = \sin u + C$$

$$7. \int \sec^2 u \, du = \tan u + C \quad 8. \int \csc^2 u \, du = -\cot u + C$$

$$9. \int \sec u \tan u \, du = \sec u + C \quad 10. \int \csc u \cot u \, du = -\csc u + C$$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)] dx$

Differential Equations**Linear Differential Equations**

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x) dx, \text{ where } \mu = e^{\int p(x) dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$) $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

Repeated Roots ($b^2 - 4ac = 0$) $y = c_1 e^{rx} + c_2 x e^{rx}$

2 Complex Roots ($b^2 - 4ac < 0$) $y = e^{ax} (c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c : complementary solution, y_p : particular solution]

Vector**Length of Vector**

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta$$

Area for parallelogram PQRS

$$= \left| \vec{PQ} \times \vec{PR} \right|$$

Area for triangle PQR

$$= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$$

Equation of LinesVector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$ Parametric equations: $x = x_0 + at$ $y = y_0 + bt$ $z = z_0 + ct$ Symmetric equation: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$ ***Equation of Planes***Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$ Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ Linear equation: $ax + by + cz + d = 0$ ***Angle between Two Planes:*** $\theta = \cos^{-1} \left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1| |\mathbf{n}_2|} \right)$